Supplementary Material

1 Statistics for different Benchmark datasets

	R	ecov	ery angulai	error	Rep	rodu	ar error	
Method	Р	σ	Median	Rank	Р	σ	Median	Rank
Gray-World [2]	-	-	7.0°	11	-	-	7.49°	11
MaxRGB [10]	-	-	6.5°	10	-	-	7.44°	10
Shades-of-gray [5]	7	-	3.7°	<u>9</u>	7	-	3.94°	8
1 st order gray-edge [14]	7	4	3.2°	7	14	4	3.59°	<u>6</u>
2 nd order gray-edge [14]	14	10	2.7°	4	15	10	3.04°	4
Pixel-based gamut [6, 8]	-	4	2.267°	2	-	4	2.832°	3
Edge-based gamut	-	2	2.278°	<u>3</u>	-	2	2.697°	2
Intersection-based gamut	-	4	2.09°	1	-	3	2.48°	1
Union-based gamut	-	2	2.95°	5	-	2	3.38°	5
Heavy tailed-based [3]	-	-	3.45°	8	-	-	4.11°	9
Weighted grav-edge [7]	2	1	3.09°	6	2	1	3.62°	7

1.1 SFU Laboratory dataset

Table 1 : Recovery and Reproduction median errors of several colour constancy algorithms for SFU Lab dataset [1]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted.

	Recovery angular error				Rep	orodu	ction angu	lar error
Method	Р	σ	Mean	Rank	Р	σ	Mean	Rank
Gray-World	-	-	9.78°	11	-	-	10.08°	11
MaxRGB	-	-	9.09°	10	-	-	9.7°	10
Shades-of-gray	6	-	6.29°	8	6	-	6.8°	8
1 st order gray-edge	1	1	6.89°	9	1	1	7.6°	9
2 nd order gray-edge	6	5	5.15°	5	6	5	5.7°	5
Pixel-based gamut	-	4	3.7°	2	-	4	4.2°	2
Edge-based gamut	-	2	3.92°	3	-	2	4.5°	3
Intersection-based gamut	-	4	3.62°	1	-	4	4.1°	1
Union-based gamut	-	3	4.55°	4	-	3	5.1°	4
Heavy tailed-based	-	-	5.63°	7	-	-	6.2°	7
Weighted grav-edge	2	1	5.48°	6	2	1	6.1°	6

Table 2: Recovery and Reproduction mean errors of several colour constancy algorithms for SFU Lab dataset [1]. Not so many changes in ranking this selection of algorithms based on the mean errors, but still there existed local changes.

	Recovery angular error					Reproduction angular error			
Method	Р	σ	95%	Rank	Р	σ	95%	Rank	
			quantile				quantile		
Gray-World	-	-	30.3°	11	-	-	27.99°	11	
MaxRGB	-	-	27.3°	10	-	-	27.25°	10	
Shades-of-gray	4	-	18.7°	9	3	-	18.92°	<u>8</u>	
1st order gray-edge	2	1	14.3°	6	2	1	15.56°	6	
2nd order gray-edge	2	2	14.2°	5	2	2	15.12°	5	
Pixel-based gamut	-	6	9.8°	1	-	7	11.12°	1	
Edge-based gamut	I	2	12.6°	3	-	2	14.35°	4	
Intersection-based gamut	-	6	9.8°	2	-	7	11.2°	2	
Union-based gamut	I	3	12.8°	4	-	3	13.2°	<u>3</u>	
Heavy tailed-based	-	-	15.9°	7	-	-	16.6°	7	
Weighted gray-edge	2	1	17.94°	8	2	1	19.3°	9	

Table 3: Recovery and Reproduction 95% quantile errors of several colour constancy algorithms for SFU Lab dataset [1]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted.

		Recovery angular error					uction angul	ar error
Method	Р	σ	Trimean ¹	Rank	Р	υ	Trimean	Rank
Gray-World	-	1	7.6°	11	-	-	8.3°	11
MaxRGB	-	1	7.5°	10	-	-	8.2°	10
Shades-of-gray	6	1	4.5°	<u>9</u>	6	-	4.8°	<u>8</u>
1st order gray-edge	8	3	3.6°	7	8	3	4.1°	<u>6</u>
2nd order gray-edge	7	5	3.3°	4	7	5	3.6°	4
Pixel-based gamut	-	4	2.5°	2	-	2	3°	2
Edge-based gamut	-	2	2.7°	3	-	2	3.2°	3
Intersection-based gamut	-	4	2.4°	1	-	3	2.76°	1
Union-based gamut	-	3	3.4°	5	-	2	3.88°	5
Heavy tailed-based	-	I	4.3°	<u>8</u>	-	-	4.96°	<u>9</u>
Weighted gray-edge	2	1	3.6°	<u>6</u>	2	1	4.3°	7

Table 4 : Recovery and Reproduction trimean errors of several colour constancy algorithms for SFU Lab dataset [1]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted.

¹ Trimean is defined as a weighted average of the data's median and its two quartiles.

1.2 Gray-ball dataset

	I	Reco	overy angula	r error	Rep	orodu	luction angular error			
Method	Р	σ	Median	Rank	Р	σ	Median	Rank		
Gray-World	I	-	6.97°	11	-	-	7.62°	11		
MaxRGB	-	-	5.30°	<u>6</u>	I	-	5.52°	5		
Shades-of-gray	8	-	5.28°	5	14	-	5.61°	<u>6</u>		
1st order gray-edge	2	1	4.64°	3	2	1	4.8°	3		
2nd order gray-edge	1	2	4.85°	4	1	2	5.1°	4		
Pixel-based gamut	-	2	5.67°	<u>9</u>	I	2	5.9°	<u>8</u>		
Edge-based gamut	-	1	5.62°	<u>8</u>	I	1	5.9°	<u>7</u>		
Intersection-based gamut	-	6	5.7°	<u>10</u>	1	2	5.9°	<u>9</u>		
Inverse intensity chromaticity space [13]	-	-	5.6°	<u>7</u>	I	I	6.1°	<u>10</u>		
Using natural image statistics [15]	-	-	3.92°	2	-	-	4.3°	2		
Exemplar-based colour constancy [9]	-	-	3.4°	1	-	-	3.7°	1		

Table 5: Recovery and Reproduction median errors of several colour constancy algorithms for Gray-ball dataset [4]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted.

]	Reco	overy angula	r error	Re	prod	uction angul	ar error
Method	Р	σ	Mean	Rank	Р	σ	Mean	Rank
Gray-World	1	I	7.87°	11	-	-	8.7°	11
MaxRGB	I	I	6.8°	7	-	-	7.1°	7
Shades-of-gray	9	I	6.11°	5	9	-	6.4°	5
1st order gray-edge	1	2	5.74°	3	1	2	6.1°	3
2nd order gray-edge	1	4	5.96°	4	1	4	6.3°	4
Pixel-based gamut	I	5	7.07 [°]	10	-	5	7.3°	10
Edge-based gamut	1	3	6.82°	8	-	3	7.1°	8
Intersection-based gamut	I	8	6.9°	9	-	7	7.2°	9
Inverse intensity chromaticity space	-	-	6.6°	6	-	-	7.03°	6
Using natural image statistics	-	-	5.2°	2	-	-	5.48°	2
Exemplar-based colour constancy	-	-	4.4°	1	-	-	4.77°	1

Table 6: Recovery and Reproduction mean errors of several colour constancy algorithms for Gray-ball dataset [4]. Not so many changes in ranking this selection of algorithms based on the mean errors, but still there existed local changes.

]	Reco	overy angula	r error	Re	prod	uction angul	angular error			
Method	Р	σ	95% quantile	Rank	Р	σ	95% quantile	Rank			
Gray-World	-	-	17.8742°	11	-	-	20.8792°	11			
MaxRGB	-	-	17.4402°	9	-	-	18.0055°	9			
Shades-of-gray	9	-	13.8419°	5	8	-	14.5032°	4			
1st order gray-edge	1	2	13.4586°	3	1	2	14.2717°	3			
2nd order gray-edge	1	3	13.8398°	4	1	4	14.6954°	5			
Pixel-based gamut	-	5	17.8235°	10	-	5	18.4689°	10			
Edge-based gamut	-	3	16.1574°	7	-	4	16.6472°	7			
Intersection-based gamut	-	9	16.1866°	8	-	8	16.9834°	8			
Inverse intensity chromaticity space	-	-	15.2475°	6	-	-	15.9256°	6			
Using natural image statistics	-	-	13.2190°	2	-	-	13.7330°	2			
Exemplar-based colour constancy	-	-	11.3178°	1	-	-	12.4545°	1			

Table 7: Recovery and Reproduction 95% quantile errors of several colour constancy algorithms for Grayball dataset [4]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted.

]	Reco	overy angula	r error	Rej	produ	action angula	ar error
Method	Р	σ	Trimean	Rank	Р	σ	Trimean	Rank
Gray-World	1	1	7.1°	11	-	-	7.87°	11
MaxRGB	I	I	5.77°	7	-	-	5.99°	<u>6</u>
Shades-of-gray	8	-	5.48°	5	12	I	5.80°	5
1st order gray-edge	1	2	5.07°	3	2	1	5.38°	3
2nd order gray-edge	1	3	5.26°	4	1	3	5.55°	4
Pixel-based gamut	-	4	6.098°	10	-	3	6.32°	10
Edge-based gamut	1	2	6.027°	8	-	1	6.3180°	8
Intersection-based gamut	1	6	6.031°	9	-	6	6.3193°	9
Inverse intensity chromaticity space	-	-	5.8°	<u>6</u>	-	-	6.23°	<u>7</u>
Using natural image statistics	-	-	4.3°	2	-	-	4.67°	2
Exemplar-based colour constancy	-	-	3.67°	1	-	-	3.97°	1

Table 8: Recovery and Reproduction trimean errors of several colour constancy algorithms for Gray-ball dataset [4]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted.

1.3 Colour-checker dataset (by Shi)

]	Recovery angular error				prod	uction angul	rr error Rank 17 16 12 14 13 2 15 3 18 11 7 5 6 4 9 9	
Method	Р	σ	Median	Rank	Р	σ	Median	Rank	
Gray-World	-	-	6.3°	17	-	-	6.8°	17	
MaxRGB	-	-	5.7°	16	-	-	6.5 [°]	16	
Shades-of-gray	5	-	3.9 [°]	12	5	-	4.4°	12	
1st order gray-edge	3	3	4.3 [°]	<u>13</u>	1	9	4.9°	<u>14</u>	
2nd order gray-edge	3	6	4.4°	<u>14</u>	1	1	4.8 [°]	<u>13</u>	
Pixel-based gamut	-	4	2.3°	2	-	4	2.7°	2	
Edge-based gamut	-	3	5.0°	15	-	3	5.8°	15	
Intersection-based gamut	-	4	2.3°	3	-	4	2.7°	3	
Regression (SVR)	-	-	6.73°	18	-	-	7.4°	18	
Bayesian	-	-	3.46 [°]	11	-	-	3.92 [°]	11	
Heavy tailed-based	-	-	2.96 [°]	<u>8</u>	-	-	3.476 [°]	<u>7</u>	
Bottom-up	-	-	2.56 [°]	5	-	-	3°	5	
Top-down	-	-	2.63 [°]	6	-	-	3.1°	6	
Bottom-up+Top-down	-	-	2.47°	4	-	-	2.8°	4	
Using Natural Image	_	_	3 13°	٩	_	_	3 5°	٩	
Statistics	_		5.15	5		_	5.5	5	
CART-based Selection	_	_	3 35°	10	_	_	3 00°	10	
[12]	_		5.55	10		_	3.50	10	
CART-based			2 91°	7	_	_	3 /179°	8	
Combination		_	2.71	<u> </u>	_	_	3.473	<u> </u>	
Exemplar-Based Color	_	_	2 27°	1	_	_	2.6°	1	
Constancy			2.21	-	_	_	2.0		

Table 9: Recovery and Reproduction median errors of several colour constancy algorithms for Colourchecker (by Shi) dataset [11]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted.

	Recovery angular error				Re	prod	uction angul	ar error
Method	Р	σ	Mean	Rank	Р	σ	Mean	Rank
Gray-World	I	I	6.4°	15	-	-	7.1°	15
MaxRGB	1	I	7.5°	17	-	-	8.1°	17
Shades-of-gray	3	I	4.9°	<u>12</u>	3	-	5.5°	<u>11</u>
1st order gray-edge	1	9	5.3°	14	1	1	6.2°	14
2nd order gray-edge	1	1	5.1°	13	1	1	6.0°	13
Pixel-based gamut	-	5	4.1°	7	-	5	4.7°	7
Edge-based gamut	I	4	6.5°	16	-	4	7.8 [°]	16
Intersection-based gamut	-	5	4.1°	8	-	5	4.7°	8
Regression (SVR)	1	I	8.08°	18	-	-	8.77°	18
Bayesian	1	I	4.82°	<u>11</u>	-	-	5.63°	<u>12</u>
Heavy tailed-based	1	I	3.67°	4	-	-	4.24°	4
Bottom-up	-	-	3.43°	2	-	-	3.98°	2
Top-down	-	-	3.75°	5	-	-	4.29°	5
Bottom-up+Top-down	-	-	3.48°	3	-	-	3.99°	3
Using Natural Image	-	-	4.19°	9	-	-	4.83°	9
CART-based Selection	-	-	4.49°	10	-	-	5.16°	10
CART-based Combination	-	-	3.9°	6	-	-	4.53°	6
Exemplar-Based Color Constancy	-	-	2.89°	1	-	-	3.4°	1

Table 10: Recovery and Reproduction mean errors of several colour constancy algorithms for Colourchecker (by Shi) dataset [11]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted.

]	Reco	overy angula	r error	Re	prod	luction angul	ar error
Method	Р	σ	95% quantile	Rank	Р	σ	95% quantile	Rank
Gray-World	-	-	11.25°	7	-	-	12.41°	<u>6</u>
MaxRGB	-	I	19.01°	18	-	-	20.05°	18
Shades-of-gray	2	I	10.56°	5	2	-	11.97°	5
1st order gray-edge	1	1	11.33°	8	1	1	14.56°	<u>11</u>
2nd order gray-edge	1	1	11.01°	<u>6</u>	1	1	13.66°	9
Pixel-based gamut	-	5	13.60°	14	-	5	15.44°	14
Edge-based gamut	-	5	16.1°	<u>16</u>	-	5	19.93°	<u>17</u>
Intersection-based gamut	-	5	13.6°	15	-	5	15.47°	15
Regression (SVR)	-	I	17.25°	<u>17</u>	-	-	18.89°	<u>16</u>
Bayesian	-	I	12.60°	13	-	-	15.39°	13
Heavy tailed-based	-	I	8.68°	2	-	-	9.89°	2
Bottom-up	-	-	9.53°	3	-	-	11.57°	4
Top-down	-	-	12.13°	<u>11</u>	-	-	13.81°	<u>10</u>
Bottom-up+Top-down	-	-	11.55°	<u>9</u>	-	-	13.59°	<u>8</u>
Using Natural Image Statistics	-	-	11.69°	<u>10</u>	-	-	12.95°	<u>7</u>
CART-based Selection	-	-	12.49°	12	-	-	14.63°	12
CART-based Combination	-	-	10.14°	<u>4</u>	-	-	11.43°	<u>3</u>
Exemplar-Based Color Constancy	-	-	6.95°	1	-	-	8.32°	1

Table 11 : Recovery and Reproduction 95% quantile errors of several colour constancy algorithms for Colour- checker (by Shi) dataset [11]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted

	Recovery angular error					prod	luction angul	ar error
Method	Р	σ	Trimean	Rank	Р	σ	Trimean	Rank
Gray-World	-	-	6.3°	16	-	-	6.9°	16
MaxRGB	-	-	6.4°	17	-	-	7.1°	17
Shades-of-gray	4	-	4.2°	<u>11</u>	4	-	4.8°	<u>12</u>
1st order gray-edge	2	6	4.7°	14	1	6	5.3°	14
2nd order gray-edge	1	1	4.6°	13	1	1	5.2°	13
Pixel-based gamut	-	4	2.9°	5	-	4	3.4°	5
Edge-based gamut	-	4	5.4°	15	-	4	6.5°	15
Intersection-based gamut	-	4	2.9°	6	-	4	3.4°	6
Regression (SVR)	-	-	7.19°	18	-	-	7.92°	18
Bayesian	-	-	4.36°	12	-	-	4.36°	<u>11</u>
Heavy tailed-based	-	-	3.11°	7	-	-	3.62°	7
Bottom-up	-	-	2.72°	3	-	-	3.15°	3
Top-down	-	-	2.81°	4	-	-	3.25°	4
Bottom-up+Top-down	-	-	2.61°	2	-	-	2.94°	2
Using Natural Image	_	_	3.45°	9	_	_	3 94°	9
Statistics			5.45				5.74	
CART-based Selection	-	-	3.55°	10	-	-	4.14°	10
CART-based	_	_	3 73°	8	_	_	3 75°	8
Combination		_	5.25	0	-	-	5.75	0
Exemplar-Based Color Constancy	-	-	2.42°	1	-	-	2.87°	1

Table 12 : Recovery and Reproduction trimean errors of several colour constancy algorithms for Colourchecker (by Shi) dataset [11]. The ranks given to each algorithm are bold and underlined if they have changed. The optimal parameters are also shown where applicable and different ones are highlighted.

2 Visualization of the worst case angular error

Suppose we image a scene under a white illuminant (the correct = $[1 \ 1 \ 1]^t$). Now we estimate the illuminate using some algorithm as $[R \ G \ B]^t$. The angle between $[R \ G \ B]^t$ and $[1 \ 1 \ 1]^t$ is the recovery error. If we take a picture of the same scene under a second illuminant $[\alpha, \beta, \gamma]^t$ then, according to the RGB model of image formation, the estimated illuminant will be $[\alpha \ *R, \beta \ *G, \gamma \ *B]^t$ (almost always illuminant estimation algorithm will return an estimate that is biased according to the same scaling factors). Now the recovery angular error is between the vectors $[\alpha, \beta, \gamma]^t$ and $[\alpha \ *R, \beta \ *G, \gamma \ *B]^t$. Following from the recovery error theorem in the paper we can solve for the illuminant which, over all possible lights, induces the maximum recovery. To visualize these 'most challenging' lights we can colour the point in the *rg* chromaticity diagram (r=R/(R+G+B) and g=G/(R+G+B)) with the colour of the worst case illuminant. This is shown in Figure 1.

The left of the figure, (A) shows the chromaticity of the initial RGB estimate (assuming the correct answer is r=1/3 and g=1/3). In the same location of the chromaticity diagram in the right panel, (B), we show the colour of the worst case light (given the initial estimate at the same location in the left panel)



Figure 1: Left panel: chromaticities of putative illuminant estimates for a scene where the correct illuminant estimate is r=1/3 and g=1/3 (white light). Right panel: given the initial wrong answer, the worst case light (the light under which a given illuminant estimation algorithm will perform worst for a given scene) is calculated.

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